Calculation of Canopy Resistance with a Recursive Evapotranspiration Model

Robert J. Lascano[A] and Steven R. Evett[B]

[A]Cropping Systems Research Laboratory, USDA-ARS, Lubbock, TX, USA, Email Robert.Lascano@ars.usda.gov
[B]Conservation and Production Laboratory, USDA-ARS, Bushland, TX, USA, Email Steve.Evett@ars.usda.gov

Abstract
The calculation of hourly and daily crop evapotranspiration (ET_c) from weather variables when using the combination method, e.g. Penman-Monteith, requires a corresponding hourly or daily value of canopy resistance (r_c). An iterative method first proposed by MI Budyko to calculate ET_c finds the surface canopy temperature (T_s) that satisfies the crop's energy balance. This method was used to calculate both hourly and daily values of r_c over a well-watered and established alfalfa crop for 26 days during the 1999-growing season in Bushland Texas. Hourly values of r_c were obtained using a graphical procedure from first, values of ET_c measured with a large weighing lysimeter; and second, from values of T_s measured with a radiometer. Results show that the daily seasonal average alfalfa r_c was 45 ± 12 s/m for both methods of calculating r_c. In the absence of a lysimeter installation to measure ET_c the measurement of T_s with a radiometer provides an alternative and practical means to calculate r_c provided that other required weather input variables are also measured over the crop canopy.

Key Words
Irrigation, crop water use

Introduction
The concept of a maximum rate of water evaporation, i.e. potential evapotranspiration (ET_p), was introduced by Thornthwaite (1944; 1948) and defined as the water loss from vegetation when a soil has at no time a deficiency of water. In general, methods to calculate ET_p are divided into empirical and theoretical; however, there is no clear distinction between the two approaches and available methods are usually a mixture of both physical considerations and empirical observations (Sibbons 1962). Theoretical methods relate the flux of water vapor from the evaporating surface to the process of turbulent diffusion, i.e. aerodynamic approach, and a second method considers the energy balance and calculates the amount of energy that is used in vaporizing water (e.g. Sutton 1953; Brutsaert 1982). A third method combines the aerodynamic and energy balance approaches eliminating the surface temperature from the relevant equations, i.e. *combination* method (Penman 1948). A similar and independent approach to calculate ET_p was given by Budyko (1951; 1956) who termed his technique the *complex* method. However, there is a major distinction between the *combination* and *complex* methods in that the assumption made by Penman (1948) regarding the linear relation between temperature and the humidity of the evaporating surface is not required by the method proposed by Budyko (1951; 1956). The Budyko method is *recursive* in that the temperature of the evaporating surface that satisfies the energy balance is found by iteration (Sellers 1964; Lascano and Van Bavel 2007; Lascano *et al.* 2009).

The application of using calculations of ET_p to estimate the amount of water used by a crop (ET_c, mm/h) was recognized by introducing a stomatal and a day-length factor (Penman 1953), which was equated to a bulk canopy resistance (r_c, s/m) term and considered in the so-called big leaf model (Monteith 1965). Methods to calculate r_c range from an explicit solution of $ET_a = f(ET_p)$ (Van Bavel and Ehrler 1968) to an empirical function of crop height and leaf area (Allen *et al.* 1989). In this paper we show how the method of Budyko (1951, 1956) hereafter, referred to as the Recursive Combination Method (RCM) and as given by Lascano and Van Bavel (2007) was used to calculate hourly and daily values of r_c for a well-watered alfalfa crop. For this purpose, two such methods on an alfalfa crop were explored and compared. First, r_c was calculated from hourly values of ET_c measured with a large weighing lysimeter and second, r_c was calculated from measured hourly values of canopy temperature (T_s, °C) measured with a radiometer.

Methods
Experimental field and weather data
In our calculations we used weather data from Bushland Texas, USA on a well-established and watered alfalfa crop. We selected 26 days, between 23 May and 12 September 1999, all after a day of irrigation and

1 – 6 August 2010, Brisbane, Australia. Published on DVD.

determined to be without equipment failure, rainfall, significant cloudiness and periods of high wind-speed. Weather data measured at Bushland Texas consisted of air (T_a, °C) and dewpoint (T_d, °C) temperature, net irradiance (R_n, W/m^2), soil heat flux (G, W/m^2), and wind-speed (U_z, m/s). Weather variables were measured every 6 s and reported as 0.5-hour averages. Surface canopy temperature (T_s, °C) was measured with a calibrated radiometer (Everest Model 4000) and details on the lysimetric measurements of alfalfa ET_c are given by Evett *et al*. (2000). Additional information on instruments and methods used to measure weather variables, G, ET_c and T_s are described by Evett (2000) and further details are given by Lascano *et al*. (2009).

Calculations

Crop ET_c is calculated from the energy balance equation of the plant canopy surface:

$$ET_c = \left(\frac{R_n + H + G}{\lambda}\right)3600$$

(1)

where λ is the latent heat of vaporization in J/kg and H is the sensible heat flux in W/m^2, and calculated as:

$$H = \frac{\rho_a C_p \left(T_a - T_s\right)}{r_a}$$

(2)

where ρ_a is the air density in kg/m^3, C_p is the specific air heat capacity in J/kg/°C, and r_a is the aerodynamic resistance in s/m calculated as given by Evett (2000):

$$r_a = \frac{ln\left[\frac{\left(z_w - d\right)}{z_{om}}\right] \; ln\left[\frac{\left(z_r - d\right)}{z_{ov}}\right]}{k^2\, U_z}$$

(3)

where z_w is the height of the wind-speed measurement in m, z_r is the measurement height for humidity in m, z_{ov} is the vapor roughness length in m, d is zero-plane displacement height in m, and k is von Karman's constant. Values of d and z_{om} were calculated as a function of crop height (h_c, m), and T_s in Eq. (2) was found iteratively with an initial value of T_s = 10 °C written in Mathcad® (*v*. 14, PTC, Needham, MA, USA) syntax by:

$$T_s = root\left[\left[R_n + G + \frac{\left(T_a - T_s\right)\times\rho_a\times C_p}{r_a} - \frac{1.323\times\frac{\exp\left(17.269\times\frac{T_s}{T_s+237.0}\right)-e_a}{T_s+237.2}}{r_a+r_c}\times\lambda\right]\times\frac{r_a}{C_p\times\rho_a}, T_s\right]$$

(4)

where *root* is a Mathcad® built-in function to solve for the value of T_s and e_a is the ambient air density in kg/m^3.

The value of r_c in Eq. (4) was calculated from measured values of ET_c and T_s, using the graphical procedure of Lascano and Van Bavel (2007). For any given hour, the alfalfa energy balance was solved from the pertinent hourly weather input (T_a, T_d, U_z, R_n and G) using the RCM. For this purpose r_c was defined as a *range* variable in Mathcad®, i.e. 10 to 70 in 5 s/m increments. Thus for each value of r_c the solution of Eq. (4) gave ET_c and T_s as a function of r_c. Then for each hour the value of ET_c measured lysimetrically and T_s measured radiometrically were used to find the corresponding value of r_c. Calculations were done for hourly values when $R_n > 0$ W/m^2.

Results and Discussion

An example of an hourly calculation of alfalfa r_c for 4 July 1999 at 14:00 hour is given in Figure 1. The weather input for this hour was R_n = 690.2 W/m^2, G = 44.4 W/m^2, T_a = 27.7 °C, T_d = 16.5 °C, and U_z = 6.9 m/s, and h_c = 0.65 m. The measured ET_c with a lysimeter was 1.05 mm, intersecting the $ET_c = f(r_c)$ at 34.2 s/m (Figure 1a) and the measured T_s with a radiometer was 28.2 °C, intersecting the $T_s = f(r_c)$ at 35.6 s/m (Figure 1b). As examples, the diurnal course of r_c from hourly measurements of ET_c and T_s for 23 May and 1

August 1999 using the procedure shown in Figure 1 are shown in Figure 2. The daily average, standard deviation and range of r_c for the 26 days of 1999 obtained from lysimetric and radiometric measurements are summarized in Table 1.

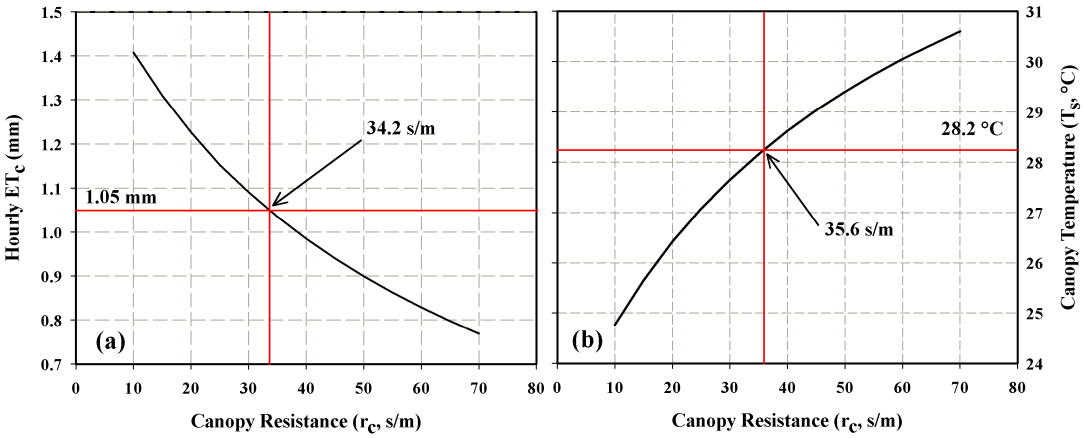

Figure 1. Graphical derivation of alfalfa canopy resistance (r_c) from (a) ET_c and (b) T_s on 4 July 1999 at 14:00 hour. The value of r_c = 34.2 s/m from the lysimeter measurement and r_c = 35.6 s/m from the radiometer measurement.

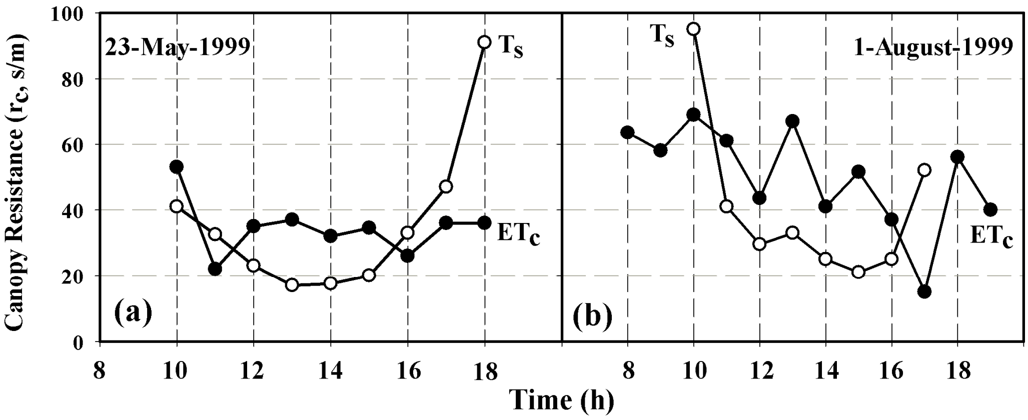

Figure 2. Diurnal values of r_c from ET_c (●) and from T_s (○) for (a) 23 May 1999 and (b) 1 August 1999 obtained using the graphical procedure shown in Figure 1.

Table 1. Daily average, standard deviation, minimum and maximum values of alfalfa r_c for 26 days 1999 in Bushland Texas, obtained from ET_c measured with a lysimeter and T_s measured with a radiometer.

	Daily canopy resistance (r_c, s/m)	
	Lysimeter (ET_c)	Radiometer (T_s)
Average	45.6	44.5
Standard Deviation	11.6	12.1
Minimum	27.0	26.3
Maximum	80.3	83.3

The graphical procedure to obtain hourly values of r_c is based on the RCM and uses an iterative solution to find T_s. This calculation does not require any more calculating effort than that used with the Penman-Monteith (PM) to find ET_c, which is based on the linearity assumption introduced by Penman (1948). However, as shown by Milly (1991) and Lascano and Van Bavel (2007) calculation of ET_c based on PM consistently underestimates evaporation, particularly under semiarid climates, where irrigation is normally used. However, a difficulty in using any model, either PM or RCM, to calculate ET_c is the selection of a r_c value that is correct.

An accurate measurement of crop ET_c can be obtained with lysimeters (Schneider *et al.* 1998); however, their construction is costly and requires continuous maintenance. An alternative is to measure the T_s of the crop with a calibrated radiometer, which can also be used to calculate r_c. Our results show that r_c can be calculated from either ET_c (Figure 1b) or T_s (Figure 1b) and that their diurnal trends are similar (Figure 2).

The largest discrepancy (data not shown) occurs early in the morning and in the evening, i.e. at low values of R_n. Nevertheless, a statistical comparison of daily averages, standard deviation and range (Table 1) suggest no differences in the seasonal value of r_c for a well-watered alfalfa derived from either hourly values of ET_c or T_s. These results suggest that in the absence of lysimeters a radiometer can be used to calculate r_c with a RCM provided that weather inputs such as, T_a, T_d, R_n, G and U_z are also measured.

Conclusion

Two methods, both based on a RCM and using a graphical procedure of hourly values of ET_c measured with a lysimeter or T_s measured with a radiometer, can be used to derive values of r_c for a well-watered alfalfa crop. Values of seasonal r_c derived from both methods were similar, 45 ± 12 s/m, which give accurate values of daily ET_c (Lascano *et al*. 2009). In the absence of a lysimetric installation to measure ET_c the measurement of T_s along with required weather variables provides an alternative method to calculate r_c that is both practical and accurate.

References

Allen RG, Jensen ME, Wright JL, Burman RD (1989) Operational estimates of evapotranspiration. *Agronomy Journal* **81**, 650-662.

Budyko MI (1951) The effect of ameliorative measures on evaporation. Izvestiia of the Academy of Sciences S.S.S.R., *Ser Geograficheskaia*, No. **1**, 16-35. (In Russian and cited from Sibbons 1962 and Milly 1991).

Budyko MI (1956) 'The Heat Balance of the Earth's Surface.' (English translation, Stepanova, N.A. Office of Technical Services, PB 131692, U.S. Department of Commerce, Washington, D.C., 1958, 259 pp.).

Brutsaert W (1982) 'Evaporation into the Atmosphere.' (Kluwer Academic Publishers, Dordrecht, The Nertherlands, 299 pp.).

Evett SR (2000) Water and energy balances at soil-plant-atmosphere interfaces. In 'Soil Physics Companion'. (Ed. AW Warrick), Chapter **5**: 127-188. (CRC-Press, Boca Raton, FL, USA).

Evett SR, Howell TA, Todd RW, Schneider AD, Tolk JA (2000) Alfalfa reference ET measurement and prediction. In 'Proceedings of the 4th Decennial National Irrigation Symposium'. Nov 14-16, 2000, Phoenix, Arizona (Ed. RG Evans, BL Benham, TP Trooien) pp. 266-272.

Lascano RJ, Van Bavel CHM (2007) Explicit and recursive calculation of potential and actual evapotranspiration. *Agronomy Journal* **99**, 585-590.

Lascano RJ, Van Bavel CHM, Evett SR (2009) A field test of recursive calculation of crop evapotranspiration. *Agricultural and Forest Meteorology* (Submitted for publication).

Milly PCD (1991) A refinement of the combination equations for evaporation. *Surveys in Geophysics* **12**, 145-154.

Monteith JL (1965) Evaporation and environment. Proceedings 19th Symposium Society Experimental Biology, Swansea, 1964. The State and Movement of Water in Living Organisms, p. 205-234.

Penman HL (1948) Natural evaporation from open water, bare soil, and grass. *Proceedings Royal Society London* **A193**, 120-145.

Penman HL (1953) The physical bases of irrigation control. *Report 13th International Horticultural Congress*: *1952* **2**, 913-924.

Schneider AD, Howell TA, Moustafa ATA, Evett SR, Abou-Zeid W (1998) A simplified weighing lysimeter for monolithic or reconstructed soils. *Applied Engineering in Agriculture* **9**, 227-232.

Sellers WD (1964) Potential evapotranspiration in arid regions. *Journal of Applied Meteorology* **3**, 98-104.

Sibbons JLH (1962) A contribution to the study of potential evapotranspiration. *Geografiska Annaler* **44**, 279-292.

Sutton G (1953) 'Micrometeorology.' (London and New York, McGraw-Hill, 302 pp.).

Thornthwaite CW (1944) Report of the committee on transpiration and evaporation, 1943-44. *Transactions, American Geophysical Union* **25**, 683-693.

Thornthwaite CW (1948) An approach toward a rational classification of climate. *Geographical Review* **38**, 55-94.

Van Bavel CHM, Ehler WL (1968) Water loss from a sorghum field and stomatal control. *Agronomy Journal* **60**, 84-86.